



Sine Waves And Hearing

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Many of us have studied the sine and cosine trigonometric functions in math and physics classes. A graph of the sine function is shown in Figure 1. In this lab, a function generator will be used to produce electrical signals with various shapes, including sine waves. The electrical signals will then be applied to an oscilloscope and a speaker, which will allow us to see, hear, and analyze the signals. The objective of this lab is to learn about the basic properties of sine waves and related signals by seeing them, hearing them, and analyzing them with an oscilloscope. We will also demonstrate some interesting facts about human hearing and speech, and the ideas will be related to the design of a telephone system.

Equipment Required

- function generator
- oscilloscope with FFT module
- microphone
- audio speaker
- Simulink software
- DSP system

1. Background About Sine Waves and Hearing

In this section, we define some terms and review some basic facts about sine waves and hearing.

1.1 Sine Waves

A *sine wave* is described by an equation of the form

$$v(t) = A \sin(2\pi ft) \quad (1)$$

where the variable t represents time. Figure 1 contains a plot of a sine wave. We use the term “wave” because the shape is similar to a water wave that you might see on an ocean or a lake. A sine wave is characterized by two parameters, called *amplitude* (A) and *frequency* (f). This lab will involve generating sine waves with various amplitudes and frequencies. Before we actually generate the sine waves, it is worthwhile to review the meaning of amplitude and frequency.

The amplitude A determines the maximum value that the sine wave achieves along the vertical axis. The sine wave takes on values between $+A$ and $-A$ at various time instants.

The frequency f of the sine wave can be understood as follows. Notice that the sine wave reaches its peak value of $+A$ at regular intervals. The time between adjacent peaks is called the *period* of the sine wave. The period is denoted by T , and it is measured in units



of seconds (sec). The frequency is defined as the number of times per second that the sine wave achieves the peak value of $+A$. Since adjacent peaks are separated by T sec, the wave achieves $1/T$ peaks per second.¹ Hence the frequency f is equal to $1/T$, and the units of frequency are sec^{-1} . Another name for the unit sec^{-1} is hertz, or Hz for short.

The amplitude and frequency of a sine wave are illustrated in Figure 1. The amplitude A determines the size of the sine wave along the vertical axis. The frequency f determines how many peaks occur per second along the horizontal axis. As the frequency increases, the peaks get closer together in time, and the wave oscillates more rapidly. We will explicitly see the effects of varying the amplitude and frequency of a sine wave later in the lab.

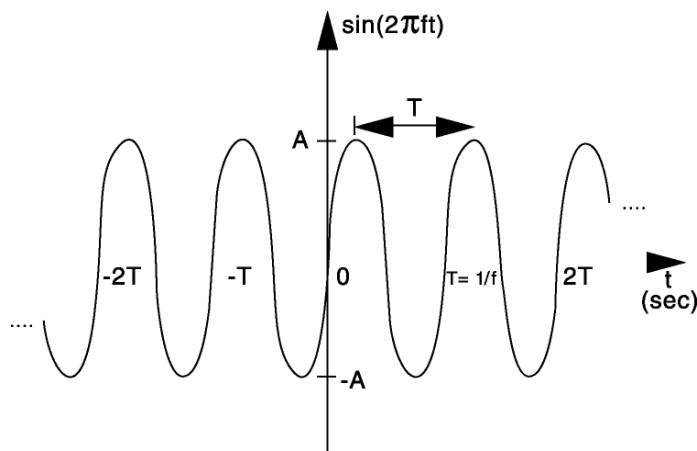


Figure 1: Sine wave with amplitude A , frequency f , and period T .

1.2 Hearing

We are exposed to a wide variety of sounds every day. We “hear” a sound after our brain processes the sensations recorded by our ears. Two attributes that are commonly used to characterize sounds are *loudness* and *pitch*. Loudness, of course, refers to how loud or intense we perceive the sound to be. Pitch refers to whether we perceive the sound to be “high” or “low”. For example, the sound of an ambulance siren has a higher pitch than the sound of a fog horn.

In this lab, we will investigate how the concepts of amplitude and frequency of a sine wave are related to the properties of loudness and pitch in hearing. Amplitude and frequency are well-defined quantities, but loudness and pitch are *subjective* quantities that are not completely understood. Reference [2] is an excellent introduction to human hearing.

The fields of psychology and human perception actually play an important role in the design of many engineering systems. An understanding of hearing is necessary to design telephones, radios, high-fidelity audio equipment, and hearing aids [1]. Similarly, engineers must understand the human visual system in order to design televisions, computer monitors, videophones, photographic equipment, FAX machines, and many other systems.



2. Lab Equipment and Setup

The equipment that will be used in this lab is illustrated in Figure 2. The *function generator* is used to create electrical signals with various shapes, including sine waves. The function generator can be programmed to generate waves with specified amplitude and frequency. The function generator is connected to two devices, an *audio speaker* and an *oscilloscope*. The speaker converts an electrical signal to sound, which we then can hear. The oscilloscope analyzes an electrical signal and displays a “picture” of the signal. The combination of the oscilloscope and the speaker allows us to “see” with our eyes what we are hearing with our ears.

Other equipment that will be used in this lab includes a microphone to convert speech to electrical signals, a software program called Simulink to build signals from sine waves, and a digital signal processor (DSP) to sample speech.

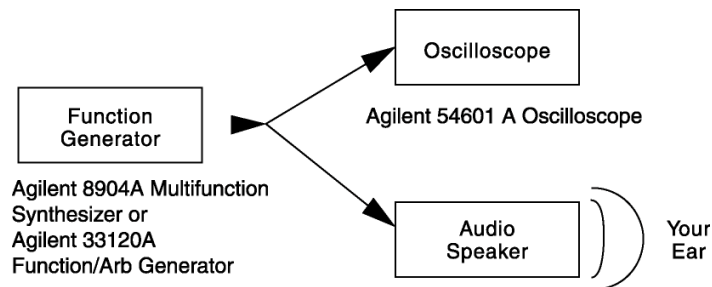


Figure 2: Configuration of laboratory equipment

3. Laboratory Procedure

This laboratory consists of a number of exercises that will first familiarize you with the instrumentation, then investigate the relationships between sine waves and hearing, and finally demonstrate how these facts are used in the design of the telephone system. The procedure for each exercise is described in the following subsections.

3.1 Measuring Amplitude and Frequency

[Note to Lab Assistant: Equipment should be pre-set. If function generator is an Agilent 33120A, set to 200mVp-p, 1kHz, output terminals = Hi Z.]

The lab equipment should already be set up so that the oscilloscope displays a sine wave. The speaker should remain off for this segment of the lab. The objective in this section is to learn how to measure the amplitude and frequency of a sine wave from the oscilloscope.

The function generator is producing a sine wave with amplitude $A = 100 \text{ mV}$ and frequency $f = 1000 \text{ Hz}$. Locate this information on the display of the function generator.

Now look at the display on the oscilloscope. Clearly, the wave has the shape of a sine function. One way to measure the amplitude and frequency of the sine wave is to use the grid on the display of the oscilloscope. This method is somewhat tedious, as some of you may know from previous experience with oscilloscopes. The oscilloscope that we are using in this lab is very convenient, because it *automatically* computes the amplitude and frequency of the sine wave. The procedure to display the amplitude of the sine wave is as follows:



1. Push the **Voltage** button on the main panel of the oscilloscope.
2. Push the “V p-p” button on the bottom of the oscilloscope display.

The peak-to-peak amplitude of the sine wave will appear in the lower left portion of the oscilloscope display. The peak-to-peak amplitude is the difference between the largest positive value of the sine wave and the largest negative value of the sine wave, so it should be nearly equal to $A - (-A) = 2A$, or 200 mV. Is the amplitude reported by the oscilloscope close to the expected value of 200 mV? (Note that the oscilloscope recomputes the sine wave amplitude from new measurements about twice per second, so the value that it reports is changing with time. The average of the measurements should be close to 200 mV.)

The frequency of the sine wave is measured by a similar procedure, which is as follows:

1. Push the **Time** button on the main panel of the oscilloscope.
2. Push the “Freq” button on the bottom of the oscilloscope display.

The frequency will appear next to the amplitude value in the lower portion of the oscilloscope display. The measured frequency should be close to the frequency of the function generator, which is 1000 Hz. Is this the case? (As with the amplitude measurement, the oscilloscope periodically forms a new estimate of the frequency. The average of the measurements should be close to 1000 Hz.)

3.2 Amplitude & Frequency vs. Loudness & Pitch

In this section, we will turn on the speaker and investigate the relationship between the amplitude and frequency of the sine wave and our perception of the sound. Begin by turning on the speaker, and adjusting the volume to a comfortable level. The sound you are hearing is usually called a *tone* with frequency 1000 Hz. Next we will vary the amplitude and frequency of the tone, and we will observe the resulting changes using the oscilloscope and our ears.

Change the amplitude of the sine wave on the function generator as follows:

1. Push the **AMPL** button on the function generator.
Note that the present amplitude is 200 mVp-p.
2. Enter the new amplitude of 100 mVp-p as follows:
 - (a) Move the <> arrows keys of the function generator until the hundreds digit is blinking.
 - (b) With the knob, change the display to 100 mVp-p.

What has changed on the oscilloscope display? What is different about the sound that you hear? Now change the amplitude back to 200 mV, using the same procedure as described above. Increase the amplitude to 300 mV, and then return it to 200 mV. What happened on the oscilloscope and to the sound when you did this? What attribute of hearing does the *amplitude* of the sine wave correspond to, loudness or pitch?



Next vary the frequency of the sine wave as follows (the amplitude should be returned to 200 mVp-p):

1. Push the **FREQ** button on the function generator. Note that the current frequency is 1000 Hz.
2. Enter the new frequency of 500 Hz as follows:
 - (a) Type FREQ-enter number-500
 - (b) Push the **Hz** key to specify the units.

What changes do you observe on the oscilloscope? What is different about the sound you hear? Now change the frequency back to 1000 Hz.² Increase the frequency to 2000 Hz, which is equivalent to 2 kHz. What happened on the oscilloscope and to the sound? What attribute of hearing does the *frequency* of the sine wave correspond to, loudness or pitch?

3.3 Human Hearing

In Section 3.2, we observed a direct correspondence between amplitude/frequency of a sine wave and loudness/pitch of the tone. The experiments in this section will point out that loudness and pitch are subjective quantities that are not related to amplitude and frequency in a simple way.

Let us investigate how our perception of loudness changes as the frequency of the sine wave is varied. Turn on the speaker, keep the sine wave amplitude fixed at 100 mV, and vary the frequency over the range from 100 Hz to 10,000 Hz. Try cycling through the following frequencies, without changing the speaker volume control: 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, and 10,000 Hz.

1. Do you notice a variation in loudness, even though the sine wave amplitude remains constant?
2. Which frequency do you hear the loudest? Is there any variation among the members of your group?
3. Generate a tone at the frequency that appears loudest. Does the pitch of this tone seem to be one that you commonly hear in speech, music, and automobile traffic?
4. Suppose you are asked to design a siren for an ambulance. Which frequency range would you choose for the pitch of the siren? Give at least two reasons why your choice of frequency for the siren is better than other frequencies.

The design of a siren is actually a challenging engineering problem. If you are interested in more details, see chapter 6 of [1].



4. Other Signal Shapes and the Telephone System

In this section, we will investigate some more complex signals that are built by adding together sine waves. As a matter of fact, any signal can be constructed by adding sine waves!

4.1 Building Signals with Simulink

[Note to Lab Assistant: Simulink Software must be set up on workstation.]

The workstation at your lab bench should be running Simulink, which is a visual programming environment for signal and system simulation. Can you add together sine waves with different frequencies and different amplitudes to produce a square wave? Can you produce a triangle wave? Which frequencies and amplitudes did you use? How many different sine waves were added?

The above exercise illustrates a theorem that was first stated by Joseph Fourier in 1806. Fourier's theorem says that *any* function can be represented as a sum of sine waves with different frequencies and properly-chosen amplitudes. Fourier analysis plays a fundamental role in many areas of science and engineering [3].

4.2 Frequency Analysis

In previous exercises, the oscilloscope was used to display sine waves as a function of time. An alternative method for analyzing and displaying signals is called the *frequency domain*. The frequency domain describes which frequencies make up a given signal. The horizontal axis in a frequency domain plot is frequency, measured in Hz. The frequency domain representation of a sine wave is very simple. It consists of a single spike that occurs at the frequency of the sine wave. An example is shown in Figure 3. Note that this spike is not perfectly sharp at 1000 Hz. This is because we can never see the entire sine wave, which exists from the infinite past ($t = -\infty$) through the infinite future ($t = +\infty$).

We can demonstrate frequency domain analysis with the oscilloscope as follows: (The speaker may be turned off during this segment.)

1. Set the function generator to produce a sine wave with amplitude 100 mV and frequency 1000 Hz.

[Note to Lab Assistant: The FFT setup must be preprogrammed on the scope. Steps: \pm Function 2 On. Function 2 MENU. Set horizontal sweep speed to 5ms/div. Push {1} (Channel 1 button) to turn off time display and show only the FFT display. Store SETUP in SETUP 2 by touching SETUP, toggling SETUP MEMORY to 2, and touching SAVE.]

2. Change the oscilloscope display from the time domain to the frequency domain by pushing the **Setup** button, then pushing the first button on the oscilloscope display until 2 appears (it cycles through the range 1-16), then push the Recall button on the display. The frequency domain plot is called an FFT, which is an acronym for the Fast Fourier Transform algorithm that is used in the computation.
3. Verify that the FFT of the sine wave is a single spike, and that this spike occurs at 1000 Hz along the frequency axis. The scale of the horizontal axis is 0 Hz at the left edge, 4.88 kHz at the center, and 9.77 kHz at the right edge.



4. Vary the amplitude and frequency of the sine wave with the function generator. How does the FFT change?

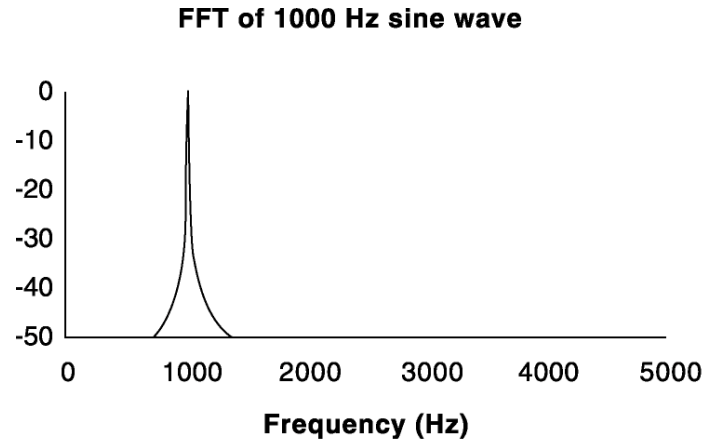


Figure 3: Frequency domain representation of a 1000 Hz sine wave.

Next use the FFT to analyze a square wave. The function generator will produce a square wave if you press the SQUARE WAVE button. (Set the amplitude to 100mV and the frequency to 1000Hz.) The wave shape is converted back to a sine by pressing the SINE WAVE button, and to a ramp and a triangle by pressing those buttons, respectively.

Does the FFT of the square wave agree with your expectations, based on section 4.1? Why or why not? Also observe the FFT of the ramp and triangle waves. You may find it interesting to [listen](#) to each shape as you observe it on the oscilloscope.

4.3 Speech Signals and Telephones

Next we will talk into a microphone and observe the speech signals on the oscilloscope.

Begin by disconnecting the function generator and speaker, and connecting the microphone to the oscilloscope. (Please ask the lab assistant or instructor for help with this.)

1. Speak into the microphone and display the corresponding electrical waveform on the oscilloscope in the time domain. You may have to press the **Autoscale** button on the oscilloscope. Can you speak (or sing or whistle) in such a way that you produce a wave that looks like a sine wave? What is the frequency of the wave?
2. Now display the speech signals on the oscilloscope in the frequency domain. Which frequencies seem to be dominant in speech? Do the dominant frequencies in speech coincide with the frequencies that are easiest for us to hear (see Section 3.3)?
3. Suppose you are to design a system for voice communication (such as the telephone system). The cost of the system increases when higher frequencies are transmitted, so it is necessary to restrict the range of frequencies. However, if too few frequencies are transmitted, then the speech becomes distorted. Given what you know about the



frequency sensitivity of human hearing and the frequencies that are dominant in speech, what range of frequencies would you transmit?

A function generator will be connected to a telephone in order to demonstrate the frequencies that are actually transmitted by the phone system. How does this limitation of the phone system affect the quality of speech and music that is transmitted?

5. Digital Signal Processing (DSP)

DSP technology is used in many products today, including compact disks for audio, telecommunications, multimedia units on computers, special effects in movies, digital television, games, automobiles and many others. The first step in any DSP system is sampling, which converts a signal such as speech or a sine wave into a list of numbers that can be processed by a computer. A key parameter in DSP systems is the sampling rate, or the number of times per second that the signal is measured. For example, music is sampled approximately 44,100 times per second to produce a compact disk.

How fast must speech be sampled for acceptable quality? This is the question to be explored in the final section of the lab.

5.1 Sampling Sine Waves

The lab assistants will help you to set up the workstations and dSPACE DSP units to sample and reconstruct a sine wave. Listen to the reconstructed signal and observe it on the oscilloscope for various sample rates and sine wave frequencies. Can you discover a relationship between the minimum allowable sampling rate and the sine wave frequency?

What sampling frequency do you think is required for a square wave? Try it out.

5.2 Sampling Speech

Next connect the microphone to the dSPACE unit instead of the function generator. Try sampling speech at various rates, and listen to the reconstructed speech on the speaker. How does the speech sound when the sampling rate is low? What seems to be the minimum acceptable sampling rate? Can you relate this rate to your observations about the frequency content of speech in Section 4.3?

6. Lab Report

The requirements for the lab report will be explained during lab. Each group should submit a “memorandum-type” lab report.



References

- [1] John G. Truxal, *The Age of Electronic Messages*. McGraw-Hill: New York, 1990.
- [2] Brian C. J. Moore, *An Introduction to the Psychology of Hearing*. Academic Press: London, 1989.
- [3] Ronald N. Bracewell, *The Fourier Transform and Its Applications*. McGraw-Hill: New York, 1986.

¹ Consider an analogy: If you turn on a light every 0.5 sec, then that is the same as turning on the light $1/0.5 = 2$ times per second.

² There are two ways to do this: either type FREQ 1000 Hz, or type FREQ 1 kHz (since 1000 Hz = 1 kHz). The latter method involves fewer keystrokes, and you may want to use it later when you enter frequencies greater than 1000 Hz.

Data Sheet

3.1) Measurements for sine wave with $A = 100$ mV and $f = 1000$ Hz:

V p-p = _____ Freq = _____

3.2) Fill in the blank with "loudness" or "pitch":

Sine wave amplitude corresponds to the _____ of the sound.

Sine wave frequency corresponds to the _____ of the sound.

3.3) Which frequencies do members of your group hear the loudest?

Which frequency range would you choose for an ambulance siren? _____

Two reasons to justify your choice:

4.1) Amplitudes used to create a *square* wave:

Amplitudes used to create a *triangle* wave:

(You may want to add more sine waves to create a better approximation.)

Also, try to duplicate your results using the *hardware* harmonic generator in lab.

4.2) How does the FFT of a sine wave change as you vary the amplitude and frequency?

How are the FFTs of the square and triangle waves related to your work in (4.1)?

4.3) Which frequencies have the largest components in speech signals?

Which frequencies are actually transmitted by the telephone system?



Can you relate this observation to the *quality* of speech on the telephone?

- 5.1) Set the sampling rate to 4000 samples/sec (or “Step size” = 1/4000). Generate sine waves with amplitude 1 V and frequencies 500, 1000, 1500, 2000, 2500, 3000, 3500, and 4000 Hz. What is the high frequency that is preserved by the sampling?

Can you describe what happens to the higher frequencies?

Can you find a general relation between the sampling rate and the highest frequency that is preserved? (You can try different sampling rates to test your relation.)

Highest Frequency Preserved = _____ x Sampling Rate.

- 5.2) Listen to speech that is sampled at the rates 10,000, 5000, and 1000 samples/sec. How does the speech sound? Does it become distorted?

Which sampling rate is used in the telephone system? _____

Explain the sampling rate used in the telephone system based on your observations about sampling sine waves in section (5.1) and your observations about the frequency content of speech in section (4.3).

Lab Reports

- One report per group. All members receive the same grade.
- Report should be typed, clearly written, and proofread.
- Use “memo-style” report as described in the EG 100 notes.
- Attach this data sheet at the end of the report. The purpose of this is to collect your data and observations so you won't have to repeat everything in the text of your report.
- Please explain the following in your report using observations from lab:
 - Relation between sine waves and hearing
 - What the FFT tells you about a signal, and conclusions you reached about speech signals based on their FFT
 - How fast a sine wave must be sampled in order to preserve its frequency
 - How the ideas in this lab are related to the telephone system. In particular, explain why the range of frequencies transmitted by telephones is reasonable, as well as why the sampling rate used in the telephone system is reasonable.
- Please describe any things that you found interesting in this lab.